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AGRICULTURAL MEMORANDUM NO. 968

Calculation of air trajectories over the land

by F B Smith and J. Gloster

INTRODUCTION

The Agricultural Section of the Meteorological Office requires a simple model for predicting the direction and speed of movement of airborne virus particles (eg Foot-and-mouth disease virus), pollen spores and insects over both land and sea. This information is needed to improve advice, both in the forecast of disease spread and in the analysis of past outbreaks, given by the Meteorological Office to, amongst others, the Ministry of Agriculture Fisheries and Food.

The Meteorological Office Boundary Layer Research Branch were asked to calculate the movement of a plume in the surface boundary layer over both the land and sea in terms of very basic, but standard, meteorological observations (the date, time, windspeed and direction measured at ten metres, the cloud amount and precipitation). Two programs have been written for a Texas Instruments TI 59 programmable calculator; the first of these is for air movement over land and the second over the sea.

This memorandum gives the formulae used in the first of these programs and the solution of these formulae for a wide range of input conditions. Appendix 1 gives the keying instructions for the program.

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Formulae used in Advection Program

The solar latitude $(\delta) = 23.3 \sin(0.9863(n-81.0))$ ..... (1)

where n = Julian day number.

The solar elevation (θ)

$$\sin\theta = S = 0.788 \sin\delta + 0.616 \cos\delta \cos(15(t-12)) \quad ...(2)$$

where t = Time (hours)

(The coefficients in (2) are $\sin\varphi$ and $\cos\varphi$ respectively when φ = the latitude = 52^0)

The Pasquill stability (P) and sensible heat flux (H) are calculated using two separate methods depending on whether $S \geq 0$ (ie daytime) or $S < 0$ (ie night time).

(a) Daytime

Net incoming solar radiation (R_N)

$$R_N = 925 S^2 - 229 S^3 - 17 - 90.8 S \quad .... (3)$$

or $$R_N = 1417 S^3 - 2600 S^2 + 1836 S - 335 \quad .... (4)$$

} See Appendix 2

Equation (3) is used when $\leq$ 6 oktas of cloud

(4) is used when $\geq$ 7 oktas of cloud

The net radiation energy is then available at the surface to drive the soil heat flux (G), the latent heat flux (LE) (ie the evaporation of moisture) and the sensible heat flux (H) into the atmosphere $R_N = G + LE + H$. The maximum solar elevation (S_{MAX}) is at t=12.

$$S_{MAX} = 0.788 \sin\delta + 0.616 \cos\delta \quad ...(5)$$

The heat available at the ground surface to go into the atmosphere $(\mathcal{R})$

$$\mathcal{R} = H + LE = R_N - G \quad .... (6)$$

$$\mathcal{R} = R_N \left(1.2 - 0.4 \frac{S}{S_{MAX}}\right) \quad .... (6a)$$ See Appendix 2

in which the flux of heat (G) to or from deeper layers of the ground has been simply parameterised in terms of S.

The sensible heat flux (H) can be derived from a formula due to Monteith (197

$$H = \frac{r_a + r_{st} - r_i}{\left(1 + \frac{\bar{\Delta}}{\gamma}\right) r_a + r_{st}} \cdot R \qquad \ldots\ldots(7)$$ See Appendix 2

where r_a = aerodynamic resistance

r_{st} = stomatal resistance

r_i = climatological resistance dependent on the relative humidity of the air.

$\bar{\Delta} = \frac{\delta e_w}{\delta T}$ the rate of change of saturated water vapour pressure with temperature averaged between the screen temperatures T and the surface temperature T_s.

γ = psychrometric constant

The dispersion of airborne substances depends on the degree of turbulence arising from surface drag and from buoyancy. The latter depends on H, whereas the former depends on the friction velocity (u_*) which is found iteratively. See appendix 2.

First estimate $$u_* = 0.058\, u_{10} \qquad \ldots\ldots(8)$$

where u_{10} is the wind speed at 10 metres ($msec^{-1}$). (Equation (8) assumes neutral stability and a surface roughness, $z_o = 0.01$ m)

This value for u_* (and subsequent estimates) are used to determine the Monin-Obukhov length scale (L)

$$L \approx -94700 \frac{u_*^3}{H} \qquad \ldots\ldots(9)$$

let $$y = \left(1 + \frac{160}{|L|}\right)^{1/4} \qquad \ldots\ldots(10)$$

and $$y_o = \left(1 + \frac{0.16}{|L|}\right)^{1/4} \qquad \ldots\ldots(11)$$

Using Businger's' standard form for the wind profile in an unstable surface layer, the next estimate of u_*

$$= 0.4\, u_{10} \Big/ \left(\ln \left\{ \frac{y-1}{y+1} \cdot \frac{y_0+1}{y_0-1} \right\} + 0.0349 \left(\tan^{-1} y - \tan^{-1} y_0 \right) \right) \quad \ldots\ldots (12)$$

Each time the new estimate of u_* is used to find, via a re-calculation of L, another value of u_*. The process is continued until a sufficiently invariant value of u_* is obtained.

Monin-Kazanski stability parameter (μ) appropriate to the overall stability of the mixing layer is

$$\mu \approx -0.038 \frac{H}{u_*^2} \quad \ldots\ldots (13)$$

again assuming a latitude of 52^0

Let $$\mu_* \equiv -\frac{\mu}{100} \quad \ldots\ldots (14)$$

The value for u_* by iteration is for a surface roughness (z_o) of 0.01 m. Pasquill stabilities are defined for a standard surface roughness, $z_o = 0.1$ m

It can be shown that $u_{*(0.1m)} \approx 1.2\, u_{*(0.01m)}$ ..... (15)

(except extreme stabilities)

Therefore $$\mu_{*(0.1m)} = \frac{0.00038 H}{(1.2\, u_*)^2} \quad \ldots\ldots (16)$$

The Pasquill stability (P) has been shown by Smith (1981) to be related to μ_* by

$$P = 3.6 \Big/ (1 + 0.53\mu_* + 4.9\mu_*^2 - 2\mu_*^3) \quad \ldots\ldots (17)$$

(b) Night time

An analysis of a rather limited amount of data at Cardington has suggested that the Pasquill stability (P) at night can be approximated by

$$P \approx \frac{3.6 + 120 - 13.3\,c}{27 - 2c}\, e^{-\frac{3}{8} u_{10}} \quad \ldots\ldots (18)$$

where c = cloud amount in oktas.

The sensible heat flux (H) is likewise given by:

$$H = \frac{\rho \times 1.8(c-q)}{(0.01 + \rho^2)} \qquad \ldots\ldots (19)$$

The provisional nature of (18) and (19) must be stressed in view of the limited data base on which rest and the likely influence of soil type, vegetation type and local topography which is not allowed for in these equations.

(c) Day time and Night time Contd

Values of the depth of the mixing layer (h), the geostrophic wind speed (G), the backing of the surface wind from the geostrophic wind (α) and the vertical standard deviation of the plume (σ_z) are required as inputs to the next part of the program. Values for h, G and α are given in Figures 1-3 as a function of the sensible heat flux (H) and the windspeed at 10 m (u_{10}) Soon after emission a plume only fills the lower part of the mixing layer. Figure 4 gives values for σ_z as a function of distance downwind of the source and Pasquill stability category. Once the plume fills the mixing layer σ_z can be approximately represented by $0.6\,h$ (this value is given in Figure 1).

The advective wind speed (U) and direction (ANG) appropriate to a plume emitted from a ground level source and travelling over typical UK agricultural countryside (with $z_o = 0.2$ m and assuming that $u_{10(z_o=0.2m)} \approx 0.7\, u_{10(z_o=0.01m)}$) .... (20) are given by :-

$$U = G - a(G - u_{10}) \qquad \text{See appendix 2} \quad .. (21)$$

$$ANG = \left(\theta_{10} - \theta'\right)\frac{h-\sigma_z}{h-10} + \left(\theta_{10} + \alpha\right)\left(1 - \frac{h-\sigma_z}{h-10}\right) \quad .. (22)$$

where θ_{10} = 10 m wind direction (degrees)

θ' = turning of the 10m wind (z_o = 1 cm) to a 20 cm z_o

θ' is almost independent of stability. In the program θ' has been taken as 9^{o}

Solution of formulae

Values of ANG and u have been calculated as a function of:-

(1) Month

(2) Time of day

(3) Cloud cover

(4) Precipitation or no precipitation

(5) Wind speed (u_{10})

In all of the calculations it has been assumed that $\sigma_z = 0.6h$ ie the plume fills the mixing layer.

(a) All months, throughout the day, $\leq$ 6/8 cloud, no precipitation, $u_{10} = 2\text{-}15$ msec^{-1}

Tables 1 and 2 give the values of ANG for all months and for January and Juen the same results are displayed pictorially (Figure 5). During the night irrespective of wind speed and time of year ANG = 11^{o}. The magnitude of ANG varied throughout the day; a minimum of 2^{o} was calculated for May and June (u_{10} = 2 msec^{-1}). The calculations suggest that u, for a given u_{10}, was practically constant throughout the year; u/u_{10} = 1.8 for u_{10} = 2 msec^{-1} and u/u_{10} = 1.6 for u_{10} = 15 msec^{-1}.

(b) All months, throughout the day, $\geq$ 7/8 cloud, no precipitation, $u_{10} = 2\text{-}15$ msec^{-1}

At night the change in cloud cover had minimal effect. During the day ANG increased on average by less than 1^{o}; this value appeared to be independent of the time of year.

(c) All months, Mid-day, $\geq$ 7/8 cloud, precipitation, $u_{10} = 2\text{-}15$ msec^{-1}

The inclusion of precipitation had a more marked affect than increasing the cloud cover alone. Figure 6 gives the increase in ANG as a function of time of year.

(d) Changes to pre-set values - r_i and r_{st}

In order to keep the program as simple and as physically small as possible it was necessary to assume average values for r_i and r_{st} These have been

varied and ANG and U re-calculated.

r_i

Table 3 shows how r_i varies as a function of temperature etc for January and June. For completeness the values assumed in the initial calculations (January = 4000/ℓ and June = 10 000/ℓ) are also given. The extreme values of r_i/ℓ given in the table, were substituted into the program and ANG and U recalculated. For Both January and June the change in ANG was less than 1^o (U_{10} = 2 $msec^{-1}$ and time = 12.00).

r_{st}

A value of r_{st} of 500 was inserted into the program and values for ANG and U re-calculated; a value of the order of 500 would be experienced for example during a very dry spell. The results are given in Table 4. It can be seen that with U_{10} = 2 $msec^{-1}$ there is about a 2^o decrease in ANG The maximum calculated difference between $ANG_{(original)} - ANG_{(500)}$ was 7^o.

Summary

A simple model which estimates the movement of a parcel of air overland has been described. This model has two main advantages over more detailed ones; firstly only standard meteorological data are required as inputs and secondly the model is physically small enough to be written for a pocket programmable calculator. The calculated values for the deflection of the mean plume wind from that of the surface wind appear to give consistent results with those obtained from more complicated models and experimental data.

References

Businger, J. A., Wyngaard, J. C., Izumi, T., and Bradley, E. F. (1971) Journal Atmospheric Science pp 181–189.

Monteith, J. L. (1975) Principals of Environmental physics, Published by Edward Arnold.

Smith, F. B. (1981) Atmospheric Environment 13 pp 879–881.

Any comments or suggestions on this paper should be referred to the authors, Meteorological Office, London Road, Bracknell, Berkshire.

May 1983

Table 1 Values of ANG (°)

JANUARY

Time Hrs	Wind speed msec^{-1} (u_{10}) 2	5	10	15
0	11	11	11	11
3	11	11	11	11
6	11	11	11	11
9	9	9	10	10
12	8	8	9	10
15	9	9	10	10
18	11	11	11	11
21	11	11	11	11

FEBRUARY

Time Hrs	Wind speed msec^{-1} (u_{10}) 2	5	10	15
0	11	11	11	11
3	11	11	11	11
6	11	11	11	11
9	9	9	10	10
12	3	5	8	10
15	9	9	10	10
18	11	11	11	11
21	11	11	11	11

MARCH

Time Hrs	Wind speed msec^{-1} (u_{10}) 2	5	10	15
0	11	11	11	11
3	11	11	11	11
6	11	11	11	11
9	4	7	9	10
12	3	5	7	9
15	4	7	9	10
18	11	11	11	11
21	11	11	11	11

APRIL

Time Hrs	Wind speed msec^{-1} (u_{10}) 2	5	10	15
0	11	11	11	11
3	11	11	11	11
6	11	11	11	11
9	4	9	10	10
12	3	5	8	9
15	4	7	10	10
18	11	11	11	11
21	11	11	11	11

MAY

Time Hrs	Wind speed msec^{-1} (u_{10}) 2	5	10	15
0	11	11	11	11
3	11	11	11	11
6	9	10	10	10
9	3	7	8	9
12	2	4	7	9
15	3	4	8	9
18	9	8	10	10
21	11	11	11	11

JUNE

Time Hrs	Wind speed msec^{-1} (u_{10}) 2	5	10	15
0	11	11	11	11
3	11	11	11	11
6	8	8	9	10
9	3	4	8	9
12	2	3	6	9
15	3	4	7	9
18	8	9	10	10
21	11	11	11	11

Table 2 Values of ANG (°)

JULY

Time Hrs	Wind speed msec^{-1} (u_{10}) 2	5	10	15
0	11	11	11	11
3	11	11	11	11
6	8	8	10	10
9	4	4	8	9
12	3	4	8	9
15	4	4	8	9
18	8	9	10	10
21	11	11	11	11

AUGUST

Time Hrs	Wind speed msec^{-1} (u_{10}) 2	5	10	15
0	11	11	11	11
3	11	11	11	11
6	8	9	10	10
9	5	6	8	9
12	3	4	8	9
15	5	6	8	9
18	9	10	10	10
21	11	11	11	11

SEPTEMBER

Time Hrs	wind speed msec^{-1} (u_{10}) 2	5	10	15
0	11	11	11	11
3	11	11	11	11
6	9	10	11	11
9	7	8	9	10
12	4	5	8	10
15	7	8	9	10
18	10	10	10	10
21	11	11	11	11

OCTOBER

Time Hrs	Wind speed msec^{-1} (u_{10}) 2	5	10	15
0	11	11	11	11
3	11	11	11	11
6	11	11	11	11
9	9	9	10	10
12	5	6	8	10
15	9	9	10	10
18	10	11	11	11
21	11	11	11	11

NOVEMBER

Time Hrs	Wind speed msec^{-1} (u_{10}) 2	5	10	15
0	11	11	11	11
3	11	11	11	11
6	10	10	11	11
9	9	9	10	10
12	9	9	10	10
15	9	9	10	10
18	10	10	11	11
21	11	11	11	11

DECEMBER

Time Hrs	wind speed msec^{-1} (u_{10}) 2	5	10	15
0	11	11	11	11
3	11	11	11	11
6	11	11	11	11
9	9	9	10	10
12	9	9	10	10
15	9	9	10	10
18	11	11	11	11
21	11	11	11	11

Table 3 Variation of r_i as a function of temperature, relative humidity and saturated water vapour pressure

Time of Year	Temp (°C)	RH (%)	e_w (mb)	r_i	Original r_i
January	4	80	6.5	2500/R	4000/R
	8	80	8.6	3400/R	
	12	80	11.2	4400/R	
June	15	60	10.3	8100/R	10,000/R
	25	60	19.0	15,000/R	
	30	60	25.5	20,000/R	

Table 4 Calculations of ANG during a very dry spell (r_i = 500)

	⩽ 6/8 cloud u_{10} = 2msec^{-1}	⩽ 6/8 cloud u_{10} = 15 msec^{-1}	⩾ 7/8 cloud u_{10} =2 msec^{-1}	⩾ 7/8 Cloud u_{10}= 15 msec^{-1}	⩾ 7/8 cloud u_{10} = 2msec^{-1} Precipitation
January	(8) 6°	(10) 7°	NC	NC	NC
March	(3) 2°	(9) 2°	NC	NC	NC
June	(2) 0°	Unknown+	(10) 3°	NC	NC
September	(4) 2°	(10) 2°	(4) 3°	(10) 3°	(10) 10°

+ Value for heat flux 320 wm^{-2} - too high to use Figures 1-3.

( ) original values

NC not calculated

Figure 1. Estimates of the depth of the mixing layer (h) and the vertical standard deviation of a plume (σ_z) [σ_z in this example = 0.6h] as a function of the sensible heat flux (H) and the windspeed at 10m (U_{10})

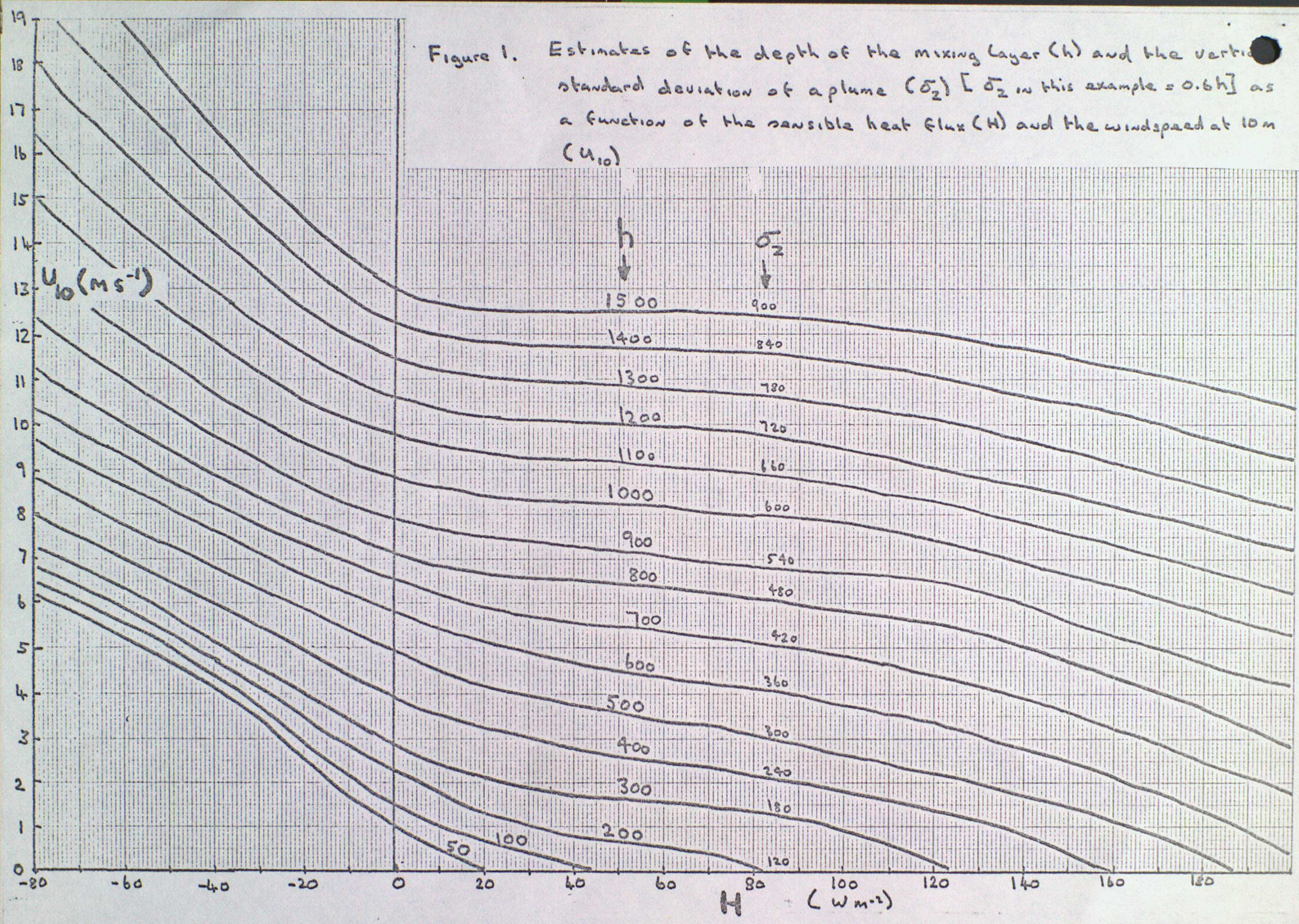


Figure 2. Estimates of the geostrophic wind (G) as a function of the sensible heat flux (H) and the windspeed at 10m (U_{10})

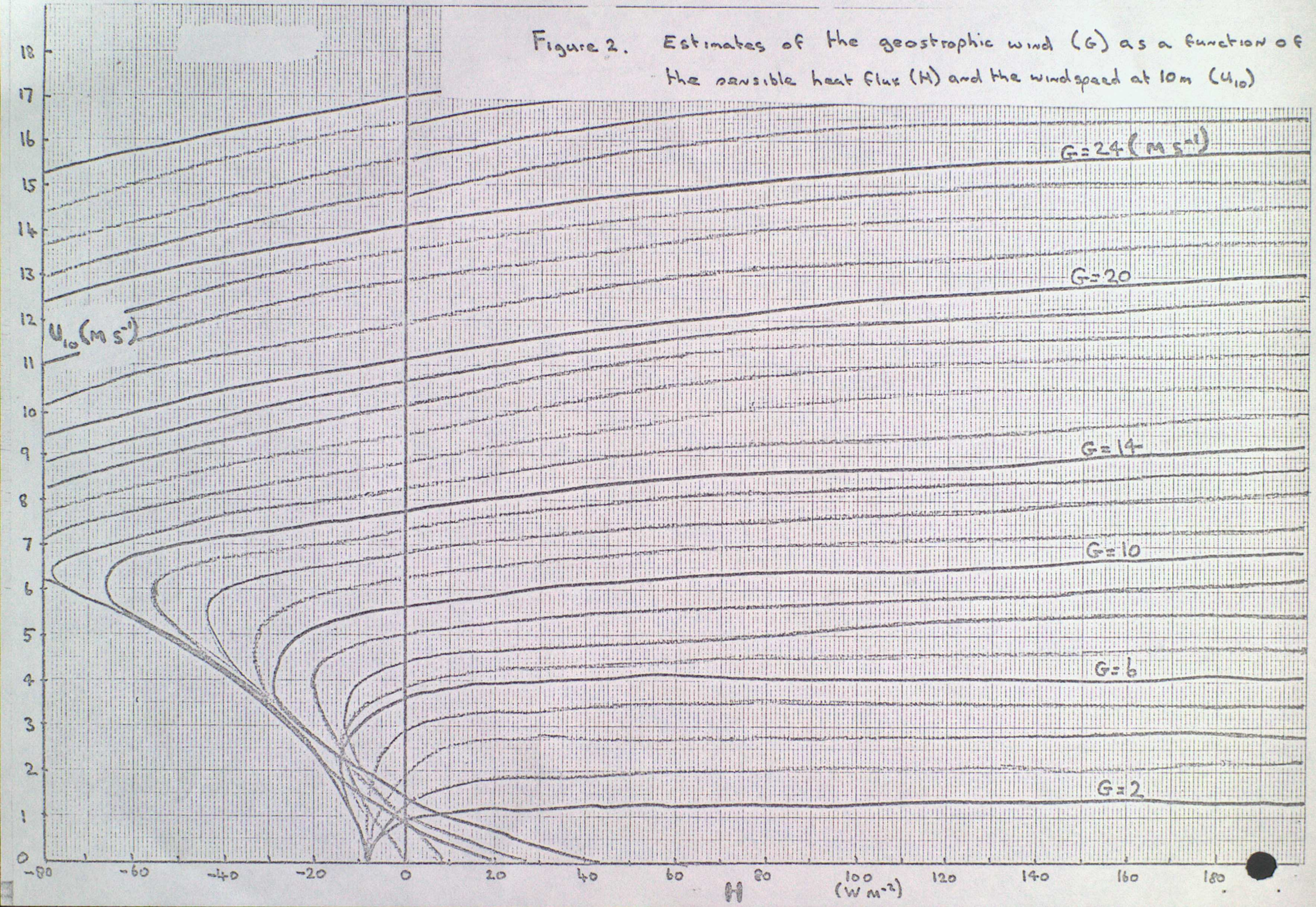


Figure 3 Estimates of the backing of the surface wind from the geostrophic wind (α) as a function of the sensible heat flux and the windspeed at 10m (U_{10})

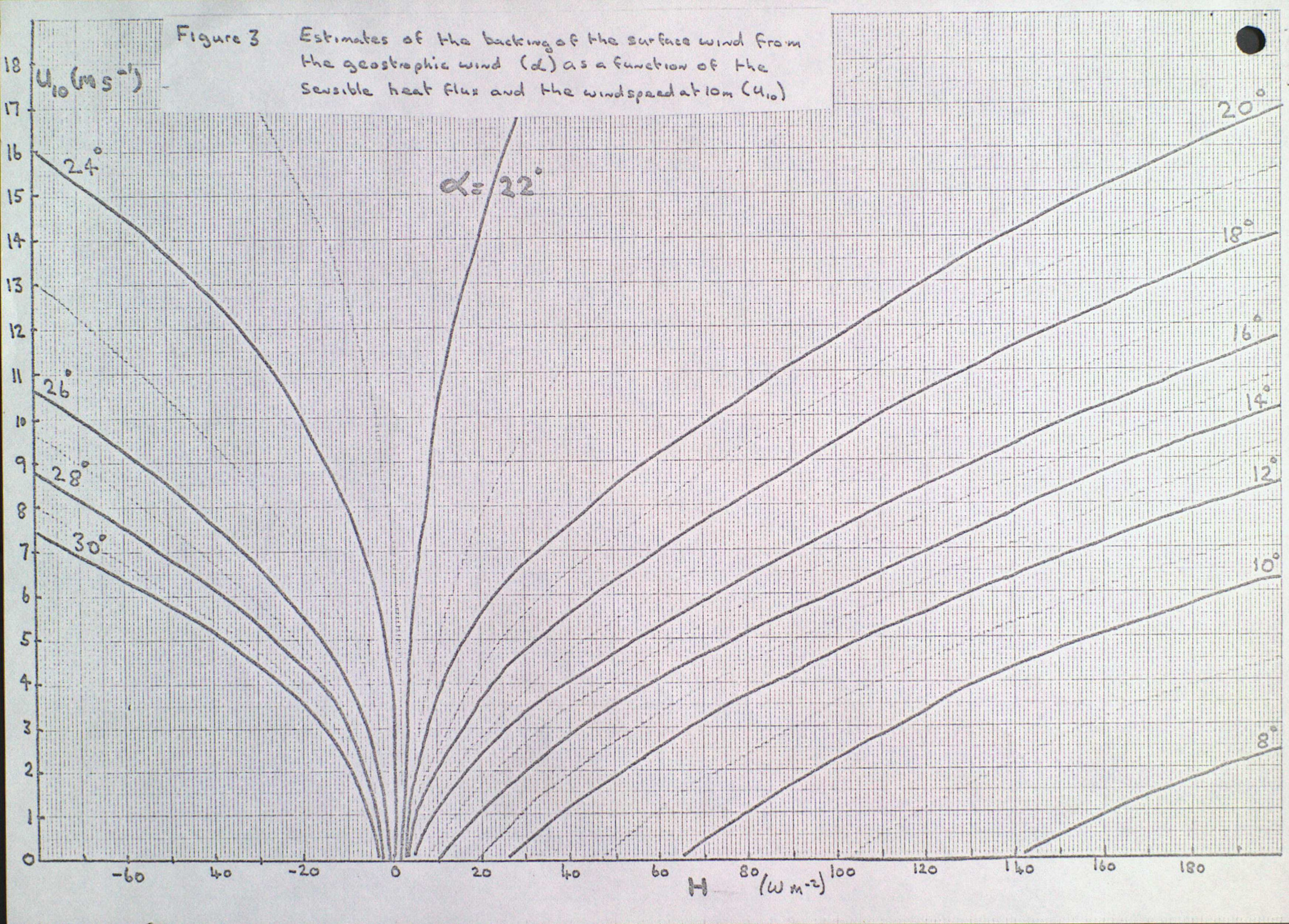


Figure 4 Vertical standard deviation of a plume (σ_z) as a function of downwind distance from the point of release and the Pasquill stability catagories (A-G or P=1 to P=6)

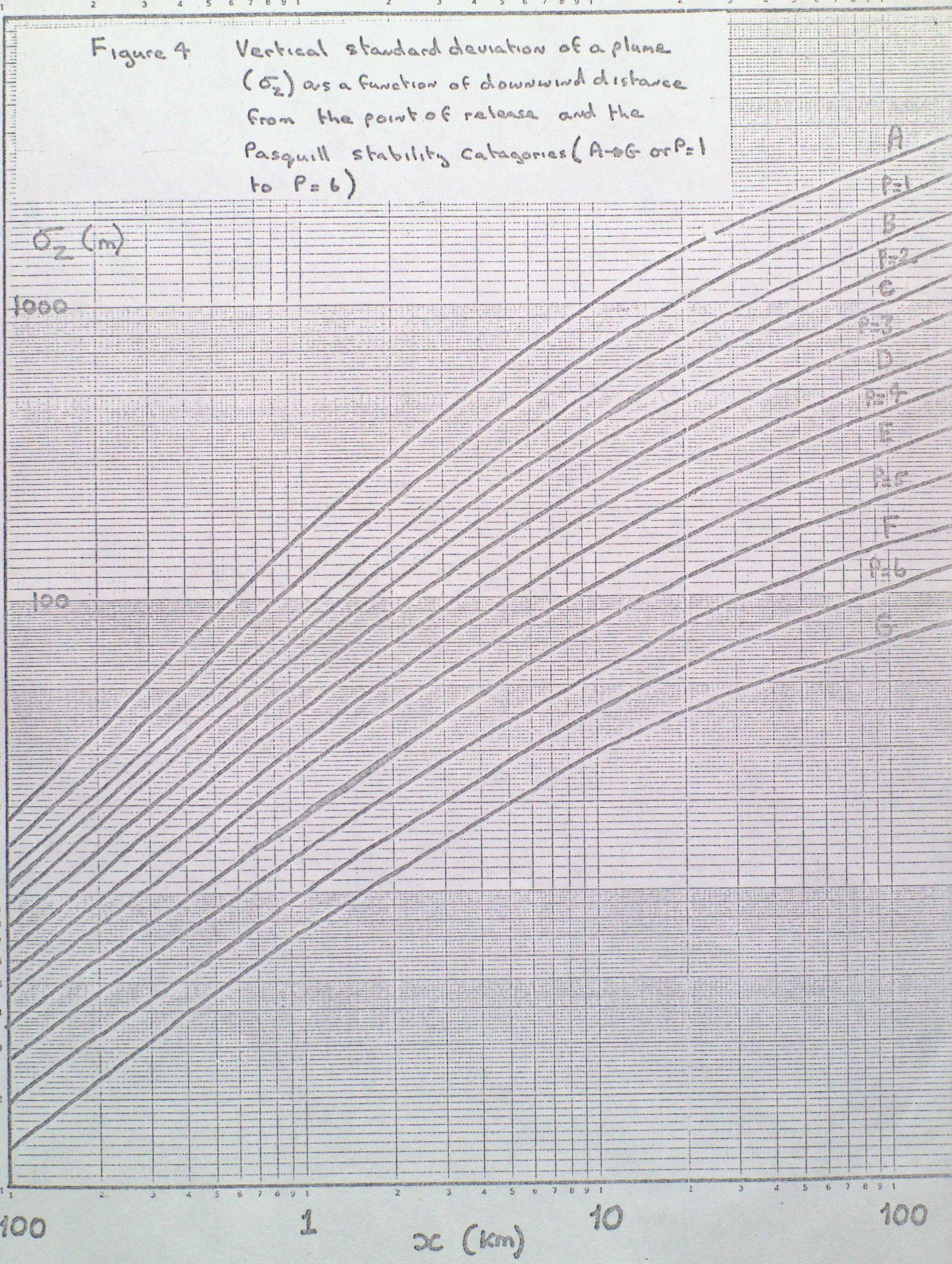


FIGURE 5. Values of ANG for January and June as a function of time of day and the windspeed at 10m (U_{10})

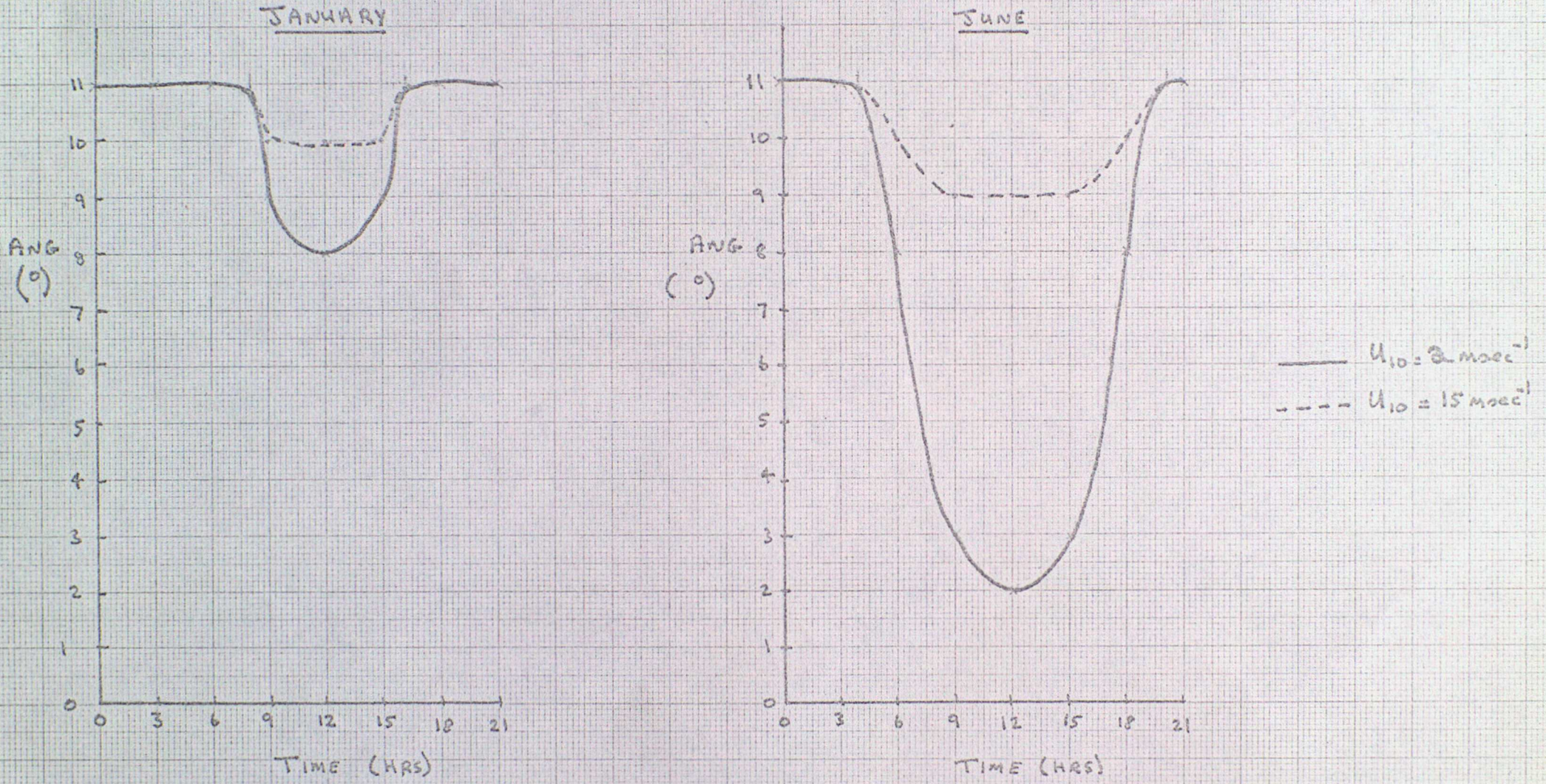
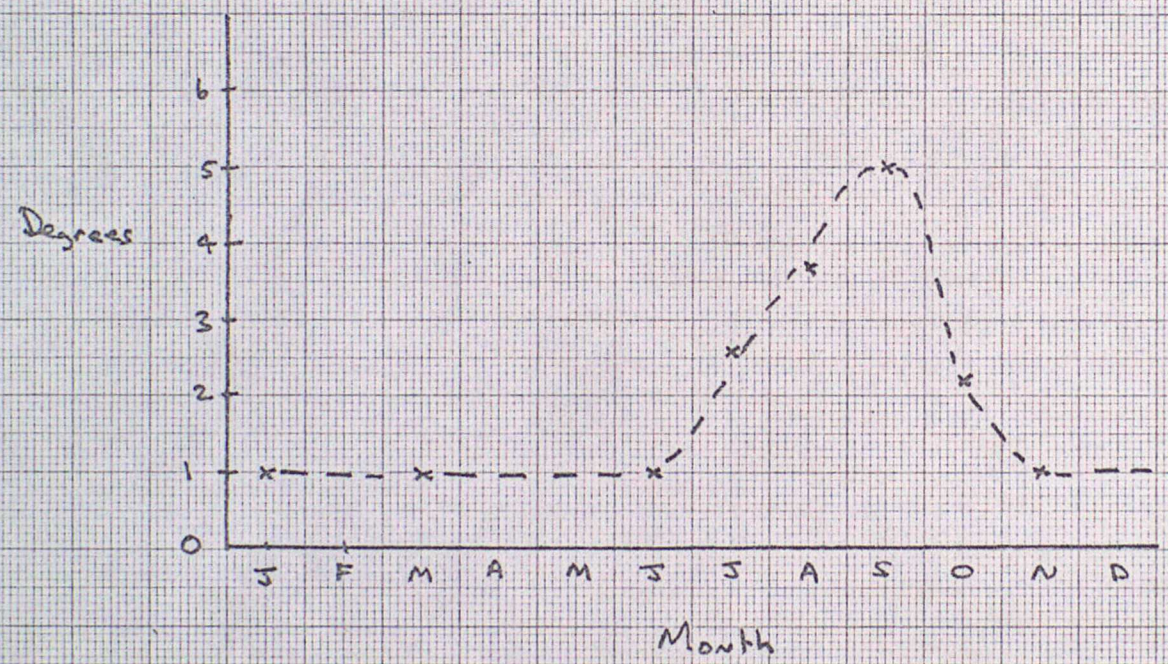


Figure 6. INCREASE OF ANG (precipitation) over NO precipitation value (Mid-day).



Appendix I Keying Instructions

Input

- x START COORD. — STO 16
- y START COORD. — STO 17
- Julian Day No. — R/S
- U_{10} — R/S
- TIME — R/S

IS VALUE DISPLAYED?

≥ 0 <0

HOW MUCH CLOUD ?

$\leq 6/8$ $\geq 7/8$

A 2nd A

2nd B

HOW MANY OCTAS OF CLOUD ?

— R/S

WHICH MONTH ?

JFM AMJ JAS OND

C 2nd C D 2nd D

R/S

IS IT RAINING ?

YES NO

GTO y^x R/S R/S

DISPLAY : INITIAL VALUE FOR U_*
ITERATE UNTIL STABLE

R/S

GTO) R/S R/S

DISPLAY : SENSIBLE HEAT FLUX
INPUT. h, G, σ_z, Surface wind direction (θ_{10}) and α

- h — R/S
- G — R/S
- σ_z — R/S
- WIND DIRECTION — R/S
- α — R/S

DISPLAY :

- CALCULATED WIND DIRECTION — R/S
- DISTANCE TRAVELLED IN 1 HOUR — R/S
- NEW x COORD. — R/S
- NEW y COORD — R/S

CLR
RST

Appendix 2 Notes on Formulae

1. Equations (3), (4) and (6a) are empirical relationships derived from analysis of Cardington data: See Smith and Blackall "Parametrization of Plumes", Mathematical Modelling of Turbulent diffusion in the environment, Edited by C J Harris, 1979 Academic Press, pp 201-236.

2. Equation(7) Monteith J. L. Principals of Environmental physics, E. Arnold Ltd, London.

r_a = aerodynamic resistance = $\frac{270}{U_{10}}$. This assumes that $z_o = 0.01$ m and a near neutral wind profile exists between 10 m and the surface.

r_i = climatological resistance = $\frac{\rho c_p}{\gamma R}\left(1 - \frac{RH}{100}\right) e_\omega(T)$

where: the specific heat at constant pressure $(c_p) = 1005\ J.kg^{-1}\ K^{-1}$ the psychrometric constant (γ) = 0.66, and ρ is the density of the air.

R = the heat available at the ground surface to go into the the atmosphere . Value calculated in Equation (6a)

RH = the relative humidity

e_ω = the saturated vapour pressure at temperature T

In the program this expression has been considerably simplified by assuming typical values for RH and T for the time of the year. The values used are:-

January, February, March	$r_i = 4000/R$
April, May, June	$r_i = 10{,}000/R$
July, August, September	$r_i = 15{,}000/R$
October, November, December	$r_i = 8{,}000/R$

r_{st} = the stomatal resistance. In the program the following values are used:-

January, February, March	$r_{st} = 80$
April, May, June	$r_{st} = 50$
July, August, September	$r_{st} = 50$
October, November, December	$r_{st} = 80$

If it is raining r_{st} is set to zero.

The term $\left(1 + \frac{\bar{\Delta}}{\gamma}\right)$ is replaced by the following approximate empirical formula: $a + b R r_a$

where a and b are a function of the time of year. The following values are used:

			a	b
January	February	March	2	2.2×10^{-5}
April	May	June	2.8	1.1×10^{-5}
July	August	September	3.2	2.25×10^{-5}
October	November	December	2.3	2.9×10^{-5}

3. The friction velocity (u_*) is found using Monin & Obukhov similarity theory. Businger (1971) Full reference see body of paper.

4. The advective wind speed (u) $= G - a(G - u_{10})$

where $a = f_2 x^2 + f_1 x$

$x \equiv f_N h/z$

f_2 and f_1 are chosen so that $a = 1$ when $u = u_{10}$

$a = \frac{G}{G - u_{10}} \equiv b$ when $u = 0$

$a = 0$ when $u = G$

$f_2 = \frac{x_0 - b x_1}{x_0 x_1 (x_1 - x_0)}$, $f_1 = \frac{1}{x_1} - f_2 x_1$

where $x_0 = f_N h/z_0$, $x_1 = f_N h/10$

$z_0 = \frac{1}{5}$ m

apply at $Z = \sigma_z$ when σ_z is the vertical standard deviation of the plume.